Close Wed: HW_5A, 5B (7.1,7.2)

Close Fri: HW_5C (7.3)

Office Hours: 1:30-3:00 in COM B-006

2. $\int_0^1 x^2 e^{x/3} dx$

7.1 Integration by Parts *(continued)*

Summary: $\int u\, dv = uv - \int v\, du$

1. Pick u = ??. The rest is dv.
2. Compute du and v.

Entry Task: Evaluate

1. $\int \frac{\ln(x)}{\sqrt{x}}\, dx$

Integration by parts is good for:

Products: xe^x, $x^2\cos(3x)$, $x\sin(5x)$

Logs: $\ln(x)$, $x^{10}\ln(x)$, $\frac{\ln(x)}{x^3}$, …

Inv. Tri: $\sin^{-1}(x)$, $\mathrm{x}\tan^{-1}(x)$, . ..

Products: $e^x\sin(x)$, $e^x\cos(x)$

Example:

$$\int \tan^{-1}(x)dx = \int \arctan(x)dx$$

Example: (Never ending integration by parts and how to end it):

$$\int e^x \cos(x)\, dx$$

7.2 Trigonometric Integral Methods

Goal: A procedure to integrate *any* combination of trig functions.

Motivating examples: ***These are substitution problems, what is u?***

$$\int \sin^3(x)\,(1-\sin^2(x))\cos(x)\,dx$$

$$\int (1-\cos^2(x))\cos^5(x)\sin(x)\,dx$$

$$\int \tan^5(x)\,(1+\tan^2(x))\sec^2(x)\,dx$$

$$\int \sec^6(x)\sec(x)\tan(x)\,dx$$

7.2 idea: Use trig identities to turn almost all trig problems into one of these situations!

Essential Tools

$$\tan(x) = \frac{\sin(x)}{\cos(x)}, \cot(x) = \frac{\cos(x)}{\sin(x)},$$
$$\sec(x) = \frac{1}{\cos(x)}, \csc(x) = \frac{1}{\sin(x)}.$$

See my online postings (or the Appendix of your book) for a more general discussion and proofs of trig identities.

$$\sin^2(x) + \cos^2(x) = 1$$
$$\tan^2(x) + 1 = \sec^2(x)$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$
$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$
$$\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$$

Case 1 (*cos(x) or sin(x) has odd power*)

$i) \int \sin^2(x)\cos^3(x)\,dx$

$ii) \int \sin^3(x)\,dx$

Case 2 (*both sin(x), cos(x) even powers*)

$i) \int \cos^2(x)\, dx$

$ii) \int \sin^4(x)\, dx$

Case 3 (*even power on sec(x)*)

$$\int \tan^2(x)\sec^4(x)\,dx$$

Case 4 (*odd power on tan(x), and at least one sec(x)*)

$$\int \tan^3(x)\sec^5(x)\,dx$$

Notes:

And if you've tried all methods and are stuck, here are things to try:

1. Rewrite in terms of sin(x) and cos(x).
2. Rewrite in terms of sec(x) and tan(x).
3. Try using trig identities.

And there are still a few "holes".
Particularly, odd power on sec(x).
For these you can quote
(proof in the book):

$$\int \tan(x)\,dx = \ln|\sec(x)| + C$$

$$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \sec^3(x)\,dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + C$$

What is the first step in each?

$$\int \sin^3(x)\cos^4(x)dx$$

$$\int \sin^5(x)\cos^3(x)dx$$

$$\int \cos^4(x)dx$$

$$\int \tan^5(x)\sec^4(x)dx$$

$$\int \tan^5(x)\sec(x)dx$$